Most of the test results of model length 3 to 4 ft reported by Millward[4] were in the preplaning range and were likely subject to tank (1.4 m wide×0.84 m deep) wall effect when wedges were attached to the planing hulls. They are not suitable for the present analysis. Instead, models of Series 62 4667-1 planing hulls of 3 ft length with larger wedge lengths were tested at the Ship Model Basin (4 m wide×2.5 m deep tank) of the National Taiwan University.[6] Figure 3 shows the test data as well as the analytical solutions obtained by solving Eqs. (8-11). In the planing range, comparisons between analytical and experimental results are reasonably good to suggest that Eqs. (4), (6), and (7) introduced in this Note could replace Eqs. (1-3) originally proposed by Brown[1] to study the effects of trim flaps or wedges on planing hulls, since Eqs. (4), (6), and (7) are applicable for both small and large flap or wedge lengths.

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Spread of Oil Slicks on a Natural Body of Water

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Introduction

THE accurate prediction of the rate of spread of oil accidentally spilled onto a natural body of water is important for initiating appropriate response measures. A number of theories exist in the literature for predicting the spread and movement of oil slicks under a variety of conditions. In particular Fay,[1] Fannelop and Waldman,[2] and Buckmaster[3] have, among others, given theoretical analyses for the spread of oil slicks on a quiescent body of water. The aforementioned analyses are based upon two restrictive assumptions. First, it is assumed that all of the oil is spilled "instantaneously," so that the total volume of the slick is conserved during the spread. Second, it is assumed that the viscous retarding force exerted on the underside of the slick by the underlying water layers can be predicted based upon concepts of laminar boundary-layer theory.

In many practical cases, such as in accidental spills at offshore drilling operations, the oil is released continuously rather than instantaneously. Moreover, flow conditions in the near-surface layers of a natural body of water are almost never laminar, so that the assumption of laminar flow to calculate the viscous force on the slick is unrealistic even when the calming effect of the slick on surface waves is taken into account. In this Note, we relax both of the foregoing restrictions and give power laws for the spread of continuous spills into a turbulent body of water. The analyses given herein are based upon simple phenomenological reasoning and on order-of-magnitude estimates of the various forces involved.

Received June 3, 1980.

Index categories: Hydrodynamics; Sea Pollution and Containment Control.

*President. Member AIAA.

Hydrodynamic Forces on the Slick

Using order-of-magnitude analyses of the forces on the slick, Fay[1] has shown that three separate regimes of spread can be identified, namely, gravity-inertial, gravity-viscous, and surface tension-viscous. In each regime the forces indicated in the pairing are assumed to be in balance, with the remaining (third) force being negligible. The points of transition from one regime of behavior to another is obtained by assuming that at three points the nonrepeating forces in two adjacent pairings are equal; that is, for example, the point of transition between the gravity-inertia and gravity-viscous regimes is found by assuming that at this point the inertia and viscous forces are equal.

Before proceeding with a description of the forces on the slick, one point of clarification regarding the nature of viscous forces on the slick is in order. The analyses given by Fannelop and Waldman[2] and by Buckmaster[3] are based upon the so-called slug-flow assumption. That is, the viscosity of the oil, relative to that of the underlying water, is assumed to be sufficiently high to insure that "the slick tends to move locally as a homogeneous slab relative to the water."[2] Consequently, the vertical gradients in the axial velocity within the slick are small, and the equations of motion can be integrated across the slick. The viscous retarding force on the slick is calculated by considering the developing *water* boundary layer below the slick, so that the force is independent of the viscosity of the *oil* itself.

For the sake of brevity, only spills into two-dimensional channels (one-dimensional spread) will be considered herein; results for spills onto open water (radial spread) follow readily and are not considered specifically. If at any time t, the length of the slick is l, then a measure of the velocity is (l/t). The different forces acting on the slick can then be estimated using order-of-magnitude analyses. If h is a measure of the thickness of the slick at time t, then we have the following estimates for the various forces (per unit width) on the slick:

Buoyancy force B

$$B \sim \rho G h^2$$

Inertia force I

$$I \sim \rho \frac{\partial u}{\partial t} \cdot lh \sim \rho \frac{l^2 h}{t^2}$$

Viscous drag force D (see Ref. 4, p. 108)

$$D \sim \rho \nu_w^{1/2} l^2 t^{-3/2}$$

Surface-tension force

$$\Sigma \sim \sigma$$

In the foregoing expressions G is the effective gravity (given by $G = g(\rho - \rho_0)/\rho$, where g is the acceleration due to gravity and ρ_0 and ρ are the densities of the oil and water, respectively), ν_w is the kinematic viscosity of water, and σ is the surface-tension spreading coefficient.[2] As in the existing theories,[1-3] the viscous drag force on the slick is calculated assuming a laminar boundary layer, so as to serve as a reference for the turbulent case to be considered presently.

Instantaneous Spills

Since in an "instantaneous" spill, the total volume of the slick is conserved, we have

$$lh = A \tag{1}$$

where A is the volume of the spill per unit channel width.

By equating the appropriate forces in each spreading regime and by using Eq. (1) we obtain the following power laws for slick growth:

Inertia-gravity regime

$$l \sim (GA)^{1/3} t^{2/3} \tag{2}$$

Gravity-viscous regime

$$l \sim (G^2 A^4 / \nu_w)^{1/8} t^{3/8} \tag{3}$$

Surface tension-viscous regime

$$l \sim (\sigma^2 / \rho^2 \nu_w)^{1/4} t^{3/4} \tag{4}$$

The foregoing power laws are the same as those derived in Refs. 1-3; they have also been summarized in Ref. 5. As noted earlier, only the viscosity of water appears in Eqs. (3) and (4).

The time scale t_1 separating the inertia-gravity and viscous-gravity regimes of slick growth can be obtained by equating I and D, and is

$$t_1 \sim (A^4 / G^2 \nu_w^3)^{1/7} \tag{5}$$

Similarly the time scale t_2 separating the viscous-gravity and surface tension-viscous regimes is obtained by equating B and Σ, and is

$$t_2 \sim (G^{1/2} A \rho \nu_w^{1/4} / \sigma)^{4/3} \tag{6}$$

Continuous Spills

For a continuous spill, Eq. (1) is no longer valid since the total volume of the oil pool is not constant in this case. If the *rate* at which the oil is spilled onto the water is assumed to be constant, then the equation corresponding to Eq. (1) is

$$lh/t = \dot{A} \tag{7}$$

where $\dot{A}$ is the rate at which the oil is spilled per unit width.

The power law for slick growth in the gravity-inertia regime for a continuous spill can be obtained by equating B and I and by using Eq. (7). Thus,

$$l \sim (G\dot{A})^{1/3} t \tag{8}$$

so that is can be seen that the leading edge of the slick will propagate into the ambient water at a constant speed. From Eqs. (7) and (8) it can also be seen that the thickness of the slick h will remain constant. The phenomenology involved herein is identical to that involved in "density currents"[6] and the leading edge of the spreading slick will assume a characteristic half-arrowhead shape. It is known[6] that in density currents the value of the densimetric Froude number based upon the velocity of the leading edge of the spreading mass and on its thickness remains constant. It can be readily verified from Eqs. (7) and (8) that the quantity $V/(Gh)^{1/2}$, where the velocity V of the leading edge is proportional to l/t, is indeed a constant independent of both G and $\dot{A}$.

It can also be verified that in the gravity-inertia regime of a continuous spill both the buoyancy and inertia forces remain constant (at their initial values B_0 and I_0, respectively); in the instantaneous-spill case, the foregoing forces decrease with increasing time. On the other hand, the viscous force D increases with time, so that a viscous-gravity regime will exist. Equating B and D, one obtains

$$l \sim (G\dot{A}^2 / \nu_w^{1/2})^{1/4} t^{7/8} \tag{9}$$

In the viscous-gravity regime, both the viscous and gravity forces *increase* with increasing time, while the inertia force decreases. Since the surface-tension force is independent of the thickness of the slick, the power law expression in the surface tension-viscous regime will be identical to Eq. (4), the expression for the instantaneous-spill case.

It can be seen by comparing Eqs. (2) and (3) with Eqs. (8) and (9) that, as would be expected on intuitive grounds, the rates of spread in the continuous-spill case are faster than those for the instantaneous-spill case. As before, the time scales t_1 and t_2 which separate the three regimes of behavior can be obtained by equating I to D and B to Σ. Thus

$$t_1 \sim (\dot{A}^4 / G^2 \nu_w^3)^{1/3} \tag{10}$$

and

$$t_2 \sim (\sigma / \rho \dot{A} G^{1/2} \nu_w^{1/4})^4 \tag{11}$$

Equations (10) and (11) should be compared with Eqs. (5) and (6), which are the corresponding results for the instantaneous-spill case.

Effects of Ambient Turbulence

Conditions in the surface layers of a natural body of water are almost always turbulent even under "calm" conditions, with the effective values of the "eddy" diffusivities in these layers being several orders of magnitude larger[7] than the molecular viscosity of water. Thus the assumption used in the literature that the viscous forces exerted on the slick by the underlying water layers can be calculated using laminar boundary-layer theory is unrealistic. If the (water) boundary layer that develops below the oil slick is turbulent, then the viscous force on the slick will be given by (see Ref. 4, p. 433)

$$\tilde{D} \sim \rho \nu_w^{1/5} l^{13/5} t^{-9/5}$$

where, as before, we have assumed $u \sim l/t$.

In the foregoing relation we have assumed that the oil/water interface remains relatively smooth, so that a laminar sublayer exists. If, on the other hand, the interface is "rough" (due to, say, wave action), then an equivalent "roughness length"[4] rather than the molecular viscosity of water will appear in the drag relation.

Clearly, the behavior of the slick in the inertia-gravity regime will remain unchanged by the ambient turbulence. The power laws for the viscous-gravity and surface tension-viscous regimes can be obtained by equating the appropriate forces, and we obtain the following results for the "instantaneous" and "continuous" spill cases:

Viscous-gravity regime for instantaneous spill case

$$l \sim (GA^2 / \nu_w^{1/5})^{5/23} t^{9/23} \tag{12}$$

Viscous-gravity regime for continuous spill case

$$l \sim (G\dot{A}^2 / \nu_w^{1/5})^{5/23} t^{19/23} \tag{13}$$

As before, the power law in the surface tension-viscous regime will be the same for both instantaneous and continuous spill cases and is

$$l \sim (\sigma / \rho \nu_w^{1/5})^{5/13} t^{9/13} \tag{14}$$

It is interesting to note that the power law indices for the turbulent flow case are not very different from the values in the corresponding laminar flow case, although the dependences on the oil and water parameters are quite different.

Conclusion

Simple power laws governing the rate of spread of oil slicks on water have been derived, after relaxing two of the restrictive assumptions made in the analyses existing in the literature, namely, that of an "instantaneous" release of the oil and that of laminar flow in the underlying water layers. The simple phenomenological and order-of-magnitude arguments used herein do *not* yield the values of the numerical coefficients in the power law relationships; the values of these coefficients have to be determined from data obtained in laboratory experiments.

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A. PUBLICATION NO.: 2 8 0 5 6 0
2. DATE OF FILING: Oct. 1, 1980

3. FREQUENCY OF ISSUE: QUARTERLY
A. NO. OF ISSUES PUBLISHED ANNUALLY: 4
B. ANNUAL SUBSCRIPTION PRICE: $5.00

4. LOCATION OF KNOWN OFFICE OF PUBLICATION *(Street, City, County, State and ZIP Code) (Not printers)*: 1290 AVENUE OF THE AMERICAS, NEW YORK, N.Y. 10104

5. LOCATION OF THE HEADQUARTERS OR GENERAL BUSINESS OFFICES OF THE PUBLISHERS *(Not printers)*: same as above

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